[Total No. of Pages : 2

236-02-05 R

M.Sc.DEGREE EXAMINATION, APRIL - 2015

Branch : Mathematics

SECOND SEMESTER

MA 205 - ADVANCED COMPLEX ANALYSIS

(Effective from the batch of students admitted in the year 2014-2015)

(Common to Applied Mathematics)

(Common to suppl. Can. also i.e, who appeared in april 2014 exam or earlier)

Time : 3 Hours

Max. Marks : 90

Part - A

Answer any **FOUR** of the following. **Each** question carries **4½** marks.

(Marks: 4×4½=18)

1. Obtain Laurent expression for $f(z)=\frac{1}{z^4(1-z)^2}$ $for|z|>1$

2. Locate the poles of $(z^2+1)^{-1}(z-1)^{-4}$ and determine their order

3. State Schwarz lemma

4. Evaluate $\int_{c_2(1)}(z^4+4)^{-1}dz$

5. State Riemann mapping theorem.

6. Prove that $u_1(x,y)=x^2-y^2, u_2(x,y)=x^3-3xy^2$ are harmonic.

7. If an entire function f(z) has no zeros, then prove that f(z) is of the form $f(z)=e^{g(z)}$ Where g(z) is an entire function

8. Prove that the infinite product $\prod_{n=1}^{\infty}(1+v_n)$ is absolutely convergent iff the infinite series $\sum_{n=1}^{\infty}v_n$ is absolutely convergent.

Part - B

Answer **All** questions. Each question carries **18**marks.

(Marks: 4×18=72)

Unit-I

9. State and prove Picard's theorem.

OR

10. a) Find the three different Laurent series representations for the function $f(z) = \frac{3}{2+z-z^2}$ involving powers of z

b) Locate the poles of $g(z) = \frac{1}{5z^4+26z^2+5}$ and specify their order

Unit-II

11. a) State and prove Cauchy residue theorem.

b) Find the residue of $f(z) = \frac{z^2+4z+5}{z^3}$ at z=0

OR

12. a) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)^3}$

b) State and prove Hurwitz theorem.

Unit-III

13. a) State and prove uniqueness theorem for conformal mapping.

b) Explain general principles of conformal mapping.

OR

14. State and prove the Dirichlet's theorem.

Unit - IV

15. a) State and prove Weierstrass theorem.

b) State and prove cauchy theorem on partial fractions.

(OR)

16. a) State and prove Mittag-Letter's theorem.

b) Expand the function secz in partial fractions.

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1. $f(z) = \frac{1-e^{2z}}{z^4}$ about the origin

2. $f(z) = \frac{1}{1-z^2}$ with centre z=1

M.Sc. DEGREE EXAMINATION, APRIL - 2015
Branch: Mathematics
SECOND SEMESTER
MA 201- GALOIS THEORY
(Effective from the batch of students admitted in the year 2014-15)
(Common for both CBCS & NON-CBCS)

Time : 3 Hours Max. Marks : 90

Part - A

Answer any Four questions. Each question carries 4½ marks

(Marks : 4×4½=18)

1. Define an irreducible polynomial *p(x) over a field F.* Show that *F(x)/(p(x))* is a field.
2. Using Eisenstein criterion, show that x^2-2 is irreducible over Q.
3. Define a splitting field of a polynomial over a field F. Give an example of a splitting field.
4. Find the degree of the extension of the splitting field of x^3-2 in Q[x].
5. With usual notation, show that $|G(E/F)| \leq [E:F]$, where E is a finite extension a field F.
6. State and prove Dedekind Lemma.
7. Show that $\phi_3(x) = x^2 + x + 1$ is cyclotomic polynomial. Find its degree.
8. Show that roots of x^4-1 in *F[x]* forms a cyclic group.

Part - B

AnswerONE question from each unit. Each question carries 18 marks.

(Marks :4×18=72)

Unit - I

9. a) State and prove Gauss lemma

   b) Let $f(x) \in f[x]$ be a polynomial of degree 2 or 3. Show that f(x) is reducible iff f(x) has a root in F.

OR

10. a) Decide $2x^5 - 5x^4 + 5$ is irreducible over Q or not.

b) Let $F \le E \le K$ be fields.If $[K:E] < \infty$ *and* $[E:F] < \infty$, *then show that* $[K:F] < \infty$.

Unit - II

11. a) Show that $Q(\alpha)$ is a normal extension of Q where $\alpha = e^{\frac{i\pi}{4}}$

b) *Is* $Q(5\sqrt{7})$ a normal extension of Q ? Justify.

OR

12. a) Show that using relevant notation $(f(x).g(x))' = f'(x).g(x) + f(x).g'(x)$

b) Let α be a root of a polynomial f(x) in $F[x]$ of degree ≥ 1. Then α is a multiple root of $f(x) \Leftrightarrow f'(\alpha) = 0$

Unit - III

13. Let E be a finite separable extension of a field F. Show that the following one equivalent.

a) E is a normal extension of F

b) $[E:F]=|G(E/F)$

(Assume relevant information)

OR

14. Show that $G(Q(\alpha)/Q)$ is isomorphic to the cyclic group of order 4 where $\alpha^5 = 1, \alpha \neq 1$

Unit - IV

15. a) Describe Klein form-group

b) Show that the Galois group of x^4+x^2+1 and Galois group of x^6-1 are same and find its order.

OR

16. Express the $x_1^2+x_2^2+x_3^2$ and $(x_1-x_2)^2(x_2-x_3)^2(x_3-x_1)^2$ as rational functions the elementary symmetric function.

[Total No. of Pages : 2 **236-02-02-R**

M.Sc. DEGREE EXAMINATION, APRIL - 2015
Branch : Mathematics
SECOND SEMESTER

MA 202-MEASURE AND INTEGRATION

(effective from the batch of students admitted in the year 2014-2015)

(Common to Applied Mathematics)

(Common to suppl. Can. also i.e., who appeared in April 2014 exam or earlier)

Time : 3 Hours Max. Marks : 90

PART - A

Answer any **FOUR** of the following. Each question carries **4½** marks

(Marks : 4×4½=18)

1. If A is a countable set then show that the set of all finite sequences from A is also countable.
2. If $A \subset B$, then prove that $\bar{A} \subset \bar{B}$ *and* $\overline{(A \cup B)} = \bar{A} \cup \bar{B}$
3. Prove that every Borel set is measurable. In particular each open set and each closed set is measurable.
4. Let $E \subset [0,1)$ be a measurable set. Prove that for each $y \in [0,1)$ the set $E \overset{0}{+} y$ is measurable and $m\left(E \overset{0}{+} y\right) = mE$
5. Let φ and ψ be a simple function which vanishes outside a set of finite measure. Prove that $\int(a\varphi + b\psi) = a\int\varphi + b\int\psi$ *and if* $\varphi \geq \psi$ *a.e then* $\int\varphi \geq \int\psi$
6. State and prove Fatou's lemma.
7. If f is integrable on $[a,b]$ *and* $\int_a^x f(t)dt = 0 \, \forall x \in [a,b]$ *then* $f(t) = 0$ *a.e in* $[a,b]$
8. If f is of bounded variation on $[a,b]$ then show that $T_a^b = P_a^b + N_a^b$ *and* $f(b) - f(a) = P_a^b + N_a^b$

PART - B

Answer all questions. Each question carries **18** marks.

(Marks : 4×18=72)

Unit - I

9. a) Show that the complement of an open set is closed and complement of a closed set is open

   b) Let a be an algebra of subsets and $\langle A_i \rangle$ a sequence of sets in $\mathfrak{a}$. Prove that there is a sequence $\langle B_i \rangle$ of sets in such that $B_n \cap B_m = \phi$ for $n \neq m$ and $\bigcup_{i=1}^{\infty} B_i = \bigcup_{i=1}^{\infty} A_i$

(Or)

10. a) State and prove Heine Borel theorem

b) Let f be a real-valued function defined on $(-\infty,\infty)$. Prove that f is continuous iff for each open set o of real numbers $f^{-1}(o)$ is an open set.

Unit - II

11. a) Prove that the interval (a,∞) is measurable

b) Let c be a constant and f and g be two measurable real valued functions defined on the same domain. Prove that $f+g, cf, fg$ are measurable

(Or)

12. a) Let E be a measurable set of finite measure, and $\langle f_n \rangle$ a sequence of measurable functions defined on E. Let f be a real valued function such that for each x in E we have $f_n(x) \to f(x)$. Then prove that for given $\epsilon > 0$ and $\delta > 0$, there is a measurable set $A \subset E$ *with* $mA < \delta$ and an integer N such that $\forall x \notin A$ *and all* $n \geq N, |f_n(x) - f(x)| < \epsilon$

b) Let $\langle E_n \rangle$ be an infinite decreasing sequence of measurable sets, i.e a sequence with $E_{n+1} \subset E_n$ for each n. Let mE_1 *be finite. Prove that* $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} mE_n$

Unit - III

13. a) State and prove bounded convergence theorem

b) Let f and g be integrable over E. Prove that

i) The function $f+g$ is integrable over E, and $\int_E f+g = \int_E f + \int_E g$

ii) If A and B are disjoint measurable sets contained in E then show that $\int_{A\cup B} f = \int_A f + \int_B f$

(Or)

14. Let f be defined and bounded on a measurable set E with mE finite. Prove that $\inf_{f\leq\psi} \int_E \psi(x)dx = Sup_{f\geq\varphi} \int_E \varphi(x)dx$ for all simple functions iff f is measurable

Unit - IV

15. a) Let f be an increasing real - valued function on the interval $[a,b]$. Show that f is differentiable almost every where. The derivative f' is measurable and $\int_a^b f'(x)dx \leq f(b) - f(a)$

b) Prove that a function f is of bounded variation on $[a,b]$ iff f is the difference two monotone real-valued functions on $[a,b]$

(Or)

16. a) If f is absolutely continuous on $[a,b]$ and $f'(x) = 0$ a.e then prove that f is constant

b) If φ is a continuous function on (a,b) and if one derivative (say D^+) of φ is non decreasing then φ is convex.

[Total No. of Pages :2 **236-02-03R**

M.Sc. DEGREE EXAMINATION, APRIL - 2015

Branch : Mathematics

SECOND SEMESTER

MA 203 - PARTIAL DIFFERENTIAL EQUATIONS

(effective from the batch of students admitted in the year 2014-15)

(Common to Applied Mathematics)

(Common to suppl. Can also i.e; who appeared in April 2014 exam or earlier)

Time : 3 Hours Max. Marks : 70

Part - A

Answer any Four questions. Each question carries 4½ marks

(Marks : 4×4½=18)

1. Find the integral curve of the equation $\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}$

2. Verify the integrability and solve the equation $x(y^2-a^2)dx + y(x^2-z^2)dy - z(y^2-a^2)dz = 0$

3. Eliminate the arbitrary function f of $f(x^2+y^2+z^2, z^2-2xy) = 0$

4. Find the general integral of the lineaf partial differential equation $(y+2x)p - (x+yz)q = x^2 - y^2$

5. Solve with usual notation $r + s - 2t = e^{x+y}$

6. If $u = f(x+iy) + g(x-iy)$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

7. If $\rho > 0$ and $\psi(r) = \int_V \frac{\rho(r')dr'}{|r-r'|}$, where V is the volume bounded. Prove that $\lim_{r\to\infty} r\psi(r) = M$, where $M = \int_V \rho(r')dr'$

8. Show that the surfaces $x^2+y^2+z^2 = cx^{2/3}$ can from a family of equipotential surfaces and find the general form of corresponding potential function.

Part - B

Answer one question from each unit. Each question carries 18 marks.

(Marks :4×18=72)

Unit - I

9. a) Find the internal curve of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$

   b) hercty the integrability of the equation and find its primitive, given $(y^2+yz)dx + (xz+z^2)dy + (y2-xy)dz = 0$

OR

10. a) Find the orthogonal trajectories on the coincoid $(x+y)z=1$ of the conics in which it is cut by the system of planes x-y+z=k, where K is a parameter

b) Prove that the necessary and sufficient condition that there exists between two functions h(x,y) and v(x,y) a relation F(u,v)=0, not involving x on y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)}=0$

Unit - II

11. a) Find the surfaces which is orthogonal & one parameter system $z=cxy(x^2+y^2)$ and which passes through the hyperbola $x^2-y^2=a^2, z=0$

b) Find the complete integral of the equation $(p^2+q^2)y=q^2$

OR

12. a) Find the general integral of linear partial differential equation $2y(z-3)p+(2x-z)q=y(2x-3)$, which passes through the circle z=0, $z^2+y+=2x$

b) Find the complete integral of $2(z+xp+yp)=yp^2$

Unit - III

13. a) Solve the equation $\frac{\partial^4 z}{\partial x^4}+\frac{\partial^4 z}{\partial y^4}=2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.

b) Reduce the equation $(n-1)^2\frac{\partial^2 z}{\partial x^2}-y^{2n}\frac{\partial^2 z}{\partial y^2}=ny^{2n-1}\frac{\partial z}{\partial y}$ to connonical form and find its general solution

OR

14. a) Solve the equation $x^2r-y^2t+xp-yq=\log x$

b) Reduce the equation $\frac{\partial^2 z}{\partial x^2}=x^2\frac{\partial^2 z}{\partial y^2}$ to canonical form.

Unit - IV

15. A uniform insulated sphere of dielectric constant k and radius a carries on its surface a charge of density $\lambda p_n(\cos\theta)$. Prove that the interior of the sphere contributes an amount

$$\frac{8\pi^2\lambda^2a^3kn}{(2n+1)(kn+n+1)^2}$$

OR

16. Prove that the solution of a certain Neumann problem can differ from one another by a constant only.

[Total No. of Pages : 2

236-02-04 R

M.Sc. DEGREE EXAMINATION, APRIL - 2015
SECOND SEMESTER
Branch : Mathematics

MA 204-TOPOLOGY

(effective from the batch of students admitted in the year 2014-2015)

(Common to Applied Mathematics)

(Common to suppl. Can. also i.e., who appeared in April 2014 exam or earlier)

Time : 3 Hours Max. Marks : 90

PART - A

Answer any **FOUR** of the following. Each question carries **4½** marks

(Marks : 4×4½=18)

1. Let X be a metric space with metric d. Show that d_1, defined by $d_1(x,y)=\frac{d(x,y)}{1+d(x,y)}$, is also a metric on X.
2. State and prove Cauchy's inequality
3. Let X be a topological space and A an arbitrary subset of X. Prove that $\bar{A}=\{x$: each neighbourhood of x intersects A$\}$
4. If f and g are real continuous functions defined on a metric space X then show that $f+g$ is also continuous.
5. Prove that any closed subspace of a compact space is compact.
6. Prove that every sequentially compact space si compact
7. Show that every compact subspace of a Hausdorff space is closed
8. Prove that a topological space is a T_1-space if and only if each point is a closed set.

PART - B

Answer **All** questions. Each question carries **18** marks.

(Marks : 4×18=72)

Unit - I

9. a) Let X be a metric space. Prove that a subset G of X is open if and only if it is a union of open spheres

   b) Let X be a complete metric space and let Y be a subspace of X, show that Y is complete if and only if it is closed.

(Or)

10. a) Let X and Y be metric spaces and f be a mapping of X in to Y. Show that f is continuous at x_0 iff $x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$

b) Prove that the set R^n of all n-tuples, $x = (x_1, x_2, \ldots x_n)$ of real numbers is a real banach space with respect to coordinate wise addition and scalar multiplication and the norm defined by $\|x\| = \left(\sum_{i=1}^{n} |x_i|^2\right)^{1/2}$

Unit - II

11. a) Prove that every separable metric space is second countable

b) Let X be a topological space. Prove that any closed subset of X is the disjoint union of its set of isolated points and its set of limit points.

(Or)

12. a) Let X be a non-empty set. Show that the family of all topologies on X is a complete lattice with respect to the relation "is weaker than".

b) Let X be a second countable space. Prove that any open base for X has a countable sub base which is also open base.

Unit - III

13. a) Prove that the product of any non-empty class of compact spaces is compact.

b) Prove that closed subspace of a complete metric space is compact if and only if it is totally bounded.

(Or)

14. a) State and prove Lebesgue's covering lemma.

b) Prove that every sequentially compact metric space is totally bounded.

Unit - IV

15. a) Prove that every compact Hausdorff space is normal

b) Let X be an arbitrary completely regular space. Prove that there exists a compact Hausdorff space $\beta(X)$ with the following properties.

i) X is a dense subspace of $\beta(X)$

ii) Every bounded continuous real function defined on X has a unique extension to a bounded continuous real function dense on $\beta(X)$

(Or)

16. State and prove Urysohn imbedding theorem.

[Total No. of Pages : 2 **B-236-02-04(a)**

M.Sc. DEGREE EXAMINATION, MAY - 2017

Branches : MATHEMATICS /APPLIED MATHEMATICS

SECOND SEMESTER

MA 204(A) : ADVANCED COMPLEX ANALYSIS

(Revised Syllabus w.e.f. 2016 - 17)

Time : 3 Hours Max. Marks : 100

Part - A

Answer any **Four** of the following questions. Each question carries **5** marks. **(4 × 5 = 20)**

1. Find the Laurent expansion of the function $f(z) = \frac{1}{z(1-z)}$ in the annulus $0 < |z| < 1$.

2. Find the singular points and investigate its behaviour at infinity for the function $\frac{e^z}{1+z^2}$.

3. Find the number of roots of the equation $z^8 - 4z^5 + z^2 - 1 = 0$ of absolute value less than 1.

4. Find the residues of $f(z) = \frac{1}{z^3 - z^5}$ at all its isolated singular points and at infinity.

5. Find the conjugate harmonic function $v(x, y)$ corresponding to $u(x, y) = x^2 - y^2 + x$ on the domain $|z| < \infty$.

6. Find the analytic function $f(z) = u(x, y) + i\, v(x, y)$ given $u(x, y) = x^2 - y^2 + 2$.

7. If an entire function $f(z)$ has no zeros, then prove that $f(z)$ is of the form $f(z) = e^{g(z)}$, where $g(z)$ is an entire function.

8. Evaluate $\prod_{n=3}^{\infty} \frac{n^2 - 4}{n^2 - 1}$.

Section - B

Answer **ALL** questions. Each question carries **20** marks. **(4 × 20 = 80)**

9. a) State and prove laurent's theorem.

b) Expand the function $f(z)=\dfrac{z^2-2z+5}{(z-2)(z^2+1)}$ on the annulus $1<|z|<2$ in a Laurent series.

OR

10. a) Prove that the point z_0 is a pole of order K of the function $f(z)$ if and only if The Laurent expansion of $f(z)$ at z_0 is of the form $f(z)=a_{-k}(z-z_0)^{-k}+\ldots + a_{-1}(z-z_0)^{-1}+a_0+a_1(z-z_0)+\ldots$; where $a_{-k}\neq 0$.

b) Prove that the function $f(z)=\sin\dfrac{1}{z}$ has an essential singular point at the origin.

11. State and prove Residue Theorem.

OR

12. a) Evaluate the integral $g=\int_{-\infty}^{\infty}\dfrac{e^{ax}}{1+e^x}dx \quad (0<a<1)$.

b) State and prove Schwarz lemma.

13. State and prove Dirichlet's problem for a disk.

OR

14. Explain Schwarz - Christoffel Transformation.

15. a) Prove that the infinite product $\prod_{n=1}^{\infty}(1+v_n)$ converges if and only if the infinite series $\sum_{n=1}^{\infty}\log(1+v_n)$ converges.

b) State and prove Weierstrass Theorem.

OR

16. State and prove mittag - Leffler's Theorem.

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B-236-02-01

M.Sc. DEGREE EXAMINATION, MAY- 2017

SECOND SEMESTER

Branch : MATHEMATICS

MA 201 : GALOIS THEORY

(Revised Syllabus w.e.f. 2016 - 17)

Time : 3 Hours

Max. Marks : 100

Section - A

Answer any **FOUR** of the following questions. Each question carries **5** marks. **(4×5=20)**

1. Let $f(x) \in F[x]$ be a polynomial of degree 2 or 3. Prove that $f(x)$ is reducible if and only if $f(x)$ has a root in F.

2. Show that $x^2 - 2$ is irreducible over Q.

3. Let F = Z/(2). Show that the splitting field of $x^3 + x^2 + 1 \in F[x]$ is a finite field with eight elements.

4. If $f(x) \in F[x]$ is irreducible over F than show that all the roots of $f(x)$ have the same multiplicity.

5. Show that the group $G(Q(\alpha)/Q)$, $\alpha^5 = 1$, $\alpha \neq 1$ is isomorphic to the cyclic group of order 4.

6. If F is a field of characteristic $\neq 2$ and $x^2 - a \in F[x]$ is an irreducible polynomial over F. Then show that its Galois group is of order 2.

7. Show that the Galois group of $x^4 + x^2 + 1$ is the same as that of $x^6 - 1$ and is of order 2.

8. Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals over Q.

Section - B

Answer **ALL** questions. Each question carries **20** marks. **(4 × 20 = 80)**

9. a) State and prove Gauss Lemma.

   b) Let $F \subseteq E \subseteq K$ be fields. If $[K : E] < \infty$ and $[E : F] < \infty$ then show that

      i) $[K : F] < \infty$

      ii) $[K : F] = [K : E] [E : F]$.

M.Sc. DEGREE EXAMINATION, DEC. 2015
FIRST SEMESTER
Branch : MATHEMATICS

MA 101 : ALGEBRA

(Under CBCS w.e.f. 2015-16)

(Common to supplementary candidates also i,e., who appeared in Nov. 2014 and earlier)

(Common to Non-CBCS)

Time : 3 Hours — Max. Marks : 90

SECTION - A

Answer any **FOUR** questions. **All** questions carry **equal** marks

(Marks : 4×4½=18)

1. Let G be a group. Show that G is a G-set with respect to the group action '*' defined by $a*x=axa^{-1}$, for all $a \in G$ and $x \in G$

2. State and prove Cayley's theorem.

3. Let $f : R \to S$ be a homomorphism of a ring R into a ring S. Then prove that ker f = {0} if and only if f is one-one.

4. If R is a ring with unity then show that each maximal ideal is prime.

5. Prove that an irreducible element in a commutative principal ideal domain (PID) is always prime.

6. Show that the ring of Gaussian integers $R = \{m + n\sqrt{-1}, m, n \in Z\}$ is a Euclidean domain.

7. State and prove schur's lemma.

8. If M is finitely generated free module over a commutative ring R, then prove that all bases of M are finite.

B-236-02-05

M.Sc. DEGREE EXAMINATION, MAY- 2017

SECOND SEMESTER

Branch : MATHEMATICS / APPLIED MATHEMATICS

MA 206 : MEASURE AND INTEGRATION

(Revised Syllabus w.e.f. 2016 - 17)

Time : 3 Hours

Max. Marks : 100

SECTION - A

Answer any **FOUR** of the following questions. Each question carries **5** marks. **(4×5=20)**

1. Let $\{A_n\}$ be a countable collection of sets of real numbers. Then prove that.

   $m^*(\cup A_n) \le \sum m^* A_n$.

2. If f is a measurable function and $f = g$ almost everywhere then prove that g is measurable.

3. Let ϕ and ψ be simple functions which vanish outside a set of finite measure. Then prove that

   $\int(a\phi + b\psi) = a\int\phi + b\int\psi$.

4. State and prove lebesgue convergence theorem.

5. If f is a bounded variation on [a, b], then prove that $T_a^b = P_a^b + N_a^b$ and $f(b) - f(a) = P_a^b - N_a^b$.

6. State and prove Jensen inequality.

7. Show that every convergent sequence is a cauchy sequence.

8. Show that $\|f + g\|_\infty \le \|f\|_\infty + \|g\|_\infty$.

SECTION - B

Answer **ALL** questions. Each question carries **20** marks. **(4×20=80)**

9. Prove that the outer measure of an interval is its length.

OR

10. a) Let A be any set, and $E_1, E_2, \ldots\ldots, E_n$ a finite sequence of disjoint measurable sets. Then prove that $m^*\left(A \cap \left[\bigcup_{i=1}^{n} E_i\right]\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$

b) Prove that the interval (α, ∞) is measureable.

OR

11. Let f be defined and bounded on a measurable set E with m E finite. In order that $\inf_{f \le \psi} \int_E \psi(x)\,dx = \sup_{f \ge \phi} \int_E \phi(x)\,dx$ for all simple functions ϕ and ψ, show that it is necessary and sufficient that f be measurable.

OR

12. a) State and prove bounded convergence theorem.

b) State and prove Fatou's lemma.

13. State and prove Vitali's lemma.

OR

14. a) Prove that *a* function f is of bounded variation on [a, b] if and only if f is the difference of two monotone real valued functions on [a, b].

b) Let f be an integrable function on [a, b] and suppose that $f(x) = f(a) + \int_a^x f(t)\,dt$. Then prove that $f'(x) = f(x)$ for almost all x in [a, b].

15. prove that L^p Spaces are Complete.

 (c)

(16) State & prove Riesz Representation th.

15. Prove that L^P spaces are complete.

OR

16. State and prove Riesz Representation Theorem.

◆◆◆◆

B-236-02-03

M.Sc. DEGREE EXAMINATION, MAY - 2017

SECOND SEMESTER

Branch : MATHEMATICS /APPLIED MATHEMATICS

MA 203 : TOPOLOGY

(Revised Syllabus w.e.f. 2016 - 17)

Time : 3 Hours

Max. Marks : 100

Section - A

Answer any **Four** of the following questions. Each question carries **5** marks. **(4 × 5 = 20)**

1. Show that in any metric space X, each open sphere is an open set.

2. Let x be a complete metric space and y be a subspace of x. Then show that y is complete if it is closed.

3. Let T_1 and T_2 be two topologies on a nonempty set x. Then show that $T_1 \cap T_2$ is also a topology.

4. Let x be a topological space and A a subset of x. Then prove that $\bar{A} = A \cup D(A)$.

5. Prove that any continuous image of a compact space is compact.

6. Prove that every compact metric space has the Bolzano - weierstross property.

7. Show that every compact subspace of a Hausdorff space is closed.

8. Prove that a subspace of the real line R is connected if it is an interval.

Section - B

Answer ALL questions. Each question carries **20** marks. **(4 × 20 = 80)**

9. a) Let X be a metric space. Prove that a subset G of x is open if and only if it is a union of open spheres. .

b) State and prove cantor's intersection theorem.

OR

10. a) Let x and y be metric spaces and of a mapping of x into y. Then prove that f is continuous at x_0 if and only if

$$x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$$

b) Prove that the set R^n of all n-tuplers $x = (x_1, x_2, ..., x_n)$ of real numbers is areal Banach space with respect to coordinate wise addition and scalar multiplication and the norm

defined by $\|x\| = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{1/2}$.

11. a) State and prove Lindelof's Theorem.

b) Let X be a second Countable space. Then prove that any open base for X has a countable subclass which is also an open base.

OR

12. a) Prove that every separable metric space is second countable.

b) Let x be a non empty set and let S be an arbitrary class of subsets of X. Then show that S can serve as an open subbase for a topology on X, in the sense that the class of all unions of finite intersections of sets in S is a Topology.

13. a) State and prove Heine - Borel Theorem.

b) State and prove Tychonoff's Theorem.

OR

14. State and Prove Ascoli's Theorem.

15. State and Prove Urysohn's lemma

OR

16. State and Prove Tietze Extension Theorem.

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